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$\sqrt{6}=\frac{2}{1}\ldots\ldots\frac{22}{9}\ldots\ldots\frac{218}{89}\ldots\ldots\frac{2158}{881}\ldots\ldots\frac{21362}{8721}\ldots\ldots\frac{211462}{86329}$.

Here is material for 15 trapeziums of the kind required.

$\sqrt{6}=\frac{5}{2}\ldots\ldots\frac{49}{20}\ldots\ldots\frac{485}{198}\ldots\ldots\frac{4801}{1960}\ldots\ldots\frac{47525}{19402}\ldots\ldots\frac{470449}{192060}\ldots\ldots\frac{4656965}{1901198}$.

These convergents furnish material for 21 trapeziums.

$\sqrt{7}=\frac{2}{1}, \frac{3}{1}, \frac{5}{2}, \frac{8}{3}, \frac{37}{14}, \frac{45}{17}, \frac{82}{31}, \frac{127}{48}, \frac{590}{223}, \frac{717}{271}, \frac{1307}{494}, \frac{2024}{765}, \frac{9403}{3554}, \frac{11427}{4319}, \frac{20830}{7873}, \frac{32257}{12192}, \frac{149858}{56641}, \frac{182115}{68833}$. In the nine odd convergents may be found the roots of 36 trapeziums. In the 2d, 6th, 10th, 14th, 18th, the roots of 10 trapeziums, and in the 4th, 8th, 12th, 16th, the seed-corn of 6 trapeziums of the kind desired.

Also solved by *G. B. M. ZERR*.

Note. Professor Zerr's solution of No. 60 is splendid, but he has left out a very difficult part of the solution, and has not referred us to the solution elsewhere. h, k, l, m, n, and p must be so taken that the sum of their fifth powers is a perfect fifth power, and I do not remember where I can find a solution of that question. *J. H. Drummond.*

MISCELLANEOUS.

59. II. **Solution by C. W. M. BLACK, A. M., Professor of Mathematics, Wesleyan Academy, Wilbraham, Mass.**

[See problem and solution I, in May number, page 156.] Consider the rays of light in a plane parallel to the axis of cylinder. It is easily seen that they are all reflected from an element of the cylinder, parallel and in another plane which together with the given plane makes equal angles with the normal plane along the reflecting element. These planes of reflection, being all perpendicular to the base, will intersect in lines perpendicular to the base, and the locus of the ultimate intersections of adjacent planes will be a caustic cylinder whose elements are perpendicular to the base of the given cylinder. As the direction of the rays of light is still oblique the intersection of the caustic cylinder with the base will be a luminous curve. The elevation of the sun does not affect the positions of the planes of light, and so does not affect the form of the caustic cylinder or of the luminous curve.

The curve is the envelope of the projection upon the base of the successive planes of light. Let circle represent base of cylinder, AB projection of incident plane, BC of reflected plane, AB being parallel to axis of X (see figure in May number, page 156.)

$OB=r$, $\angle ABO=\angle CBO=\angle BOK=\alpha$, $BKX=2\alpha$. Equation to BK is $y=(x-OK)\tan 2\alpha=(x-\frac{1}{2}r\sec\alpha)\tan 2\alpha$.

Let $\tan\alpha=a$. Then $y=[x-\frac{1}{2}r\sqrt{(1+a^2)}][(2a)/(1-a^2)]$ ....(1).

Differentiating with reference to a, equating to zero, and clearing of fractions, we have

$$(a^2-1)a^2r+2x(1+a^2)^{\frac{3}{2}}-r(1+a^2)^2=0\,;$$

whence $x/r=(1+3a^2)/[2(1+a^2)^{\frac{3}{2}}]$. Substituting in (1),

$$y/r=\{(1+3a^2)/[2(1+a^2)^{\frac{3}{2}}]-[(1+a^2)^{\frac{1}{2}}/2]\}[(2a)/(1-a^2)]=a^3/(1+a^2)^{\frac{3}{2}}\,.$$

$$\therefore\ (y/r)^{\frac{1}{3}}=a/(1+a^2)^{\frac{1}{2}}\,;\ (y/r)^{\frac{1}{3}}=a^2/(1+a^2).$$

$$a^2=(y/r)^{\frac{2}{3}}/[1-(y/r)^{\frac{2}{3}}];\ \sqrt{(1+a^2)}=1/\sqrt{[1-(y/r)^{\frac{2}{3}}]}\ \ldots\ldots\ldots\ldots(2).$$

$$1-a^2=[1-2(y/r)^{\frac{2}{3}}]/[1-(y/r)^{\frac{2}{3}}].$$

$$a/(1-a^2=(y/r)^{\frac{1}{3}}\sqrt{[1-(y/r)^{\frac{2}{3}}]}/[1-2(y/r)^{\frac{2}{3}}] \ldots\ldots\ldots\ldots\ldots\ldots(3).$$

Substitute (2) and (3) in (1),

$$y=\left(x-\frac{r}{2\sqrt{[1-(y/r)^{\frac{2}{3}}]}}\right)\times\frac{2(y/r)^{\frac{1}{3}}\sqrt{[1-(y/r)^{\frac{2}{3}}]}}{1-2(y/r)^{\frac{2}{3}}};$$

$$x=\frac{y[1-2(y/r)^{\frac{2}{3}}]}{2(y/r)^{\frac{1}{3}}\sqrt{[1-(y/r)^{\frac{2}{3}}]}}+\frac{r}{2\sqrt{[1-(y/r)^{\frac{2}{3}}]}},$$

or $x=\frac{1}{2}(r^{\frac{2}{3}}+2y^{\frac{2}{3}})\sqrt{(r^{\frac{2}{3}}-y^{\frac{2}{3}})}$, the equation sought.

60. Proposed by S. HART WRIGHT, M. D., A. M., Ph. D., Penn Yan, N. Y.

When the sun's declination is 23° 27′ 15″ North$=\delta$, in what latitude will it shine on the *north* side of buildings during the first half of the forenoon, and on the *south* side during the other half, and what will be the length of the day ?

I. Solution by the PROPOSER.

Let P be the north pole, PA a meridian, Z the zenith, HS the horizon, the sun rising at S, and describing the small circle arc SBA, during the forenoon, reaching A at apparent noon, ZH a prime vertical, bisecting the semi-diurnal arc SBA at B, and right-angled at Z, the declination$=\delta$, and latitude$=\lambda$, $\tan\delta=\alpha$, and $\tan\lambda=\chi$.

Then $PB=90°-\delta$, $PZ=90°-\lambda$, and $\angle ZPB=h=$the hour-angle at the pole, and measures the time of describing the arc BA on the south side of ZH.

$\cos h=\tan\delta\cot\lambda=\alpha\chi$.

The hour angle measuring $SBA=2h$, and $\cos 2h=-\tan\delta\tan\lambda=-\alpha\chi$. But $\cos 2h=(2\alpha^2/\chi^2)-1=-\alpha\chi$. $\therefore\ \chi^3-(\chi^2/\alpha)+2\alpha=0$. Solving this cubic, gives a positive root $\chi=2.1099$, and $\lambda=64°\ 38'\ 30''$.

$\therefore\ h=156°\ 16'\ 4''=10$ hours, 25 minutes, 4 seconds, and $2h=20$ hours, 50 minutes, 8 seconds, length of day.

II. Solution by JOHN M. ARNOLD, Crompton, R. I.

In the triangle ABC formed by the zenith, the pole, and the sun when crossing the prime vertical, $A=90°$, $B=$the hour angle, $a=$sun's polar distance, and $c=$the co-latitude.

In the triangle $A'B'C'$ formed by the north point of the horizon, the pole, and the sun when rising, $A'=90°$, $B'=$supplement of the hour angle at rising, $c'=$the latitude, and $a'=a=90°-\delta=66°\ 32'\ 45''$.

From trigonometry, $\cos B=\cot a\tan c$......(1) ; $\cos B'=\cot a'\tan c'$ .....(2) ; $c'+c=90°$....(3). From the conditions of the problem, $B'=180°-2B$....(4). From (2) and (3), $\cos B'=\cot a\cot c$. Eliminating $\cot c$ by (1), $\cos B'=\cot^2 a/\cos B$. Eliminating $\cos B'$ by (4), $2\cos^2 B-1=-(\cot^2 a/\cos B)$. Putting $\cos B=y$ and substituting the numerical value for $\cot^2 a$, $y^3=\frac{1}{2}y+.094118=0$. Finding the two positive roots of this equation, $y=.5815$ and $.2056$. From the first root $B=54°\ 27'=3$ hours, 37 minutes, 48 seconds. Length of day$=4B=14$ hours,